SIMRES-TV: NOISE AND RESIDUAL SIMILARITY FOR PARAMETER ESTIMATION IN TOTAL VARIATION

V. B. Surya Prasath1,2,3,4, Nguyen Ngoc Hien5, Dang N. H. Thanh6,*; Sergey Dvoenko7

1 Division of Biomedical Informatics, Cincinnati Children’s Hospital Medical Center, Cincinnati, OH 45229 USA
2 Department of Pediatrics, University of Cincinnati College of Medicine, Cincinnati, OH 45257, USA
3 Department of Biomedical Informatics, College of Medicine, University of Cincinnati, OH 45267 USA
4 Department of Electrical Engineering and Computer Science, University of Cincinnati, OH 45221 USA - prasatsa@uc.edu
5 Dong Thap University, Cao Lanh City, Vietnam - nguyennghoched@dhu.edu.vn
6 Department of Information Technology, School of Business Information Technology, University of Economics Ho Chi Minh City, Vietnam - thanhdinh@ueh.edu.vn
7 Institute of Applied Mathematics and Computer Science, Tula State University, Russia - dvsrg@gmail.com

Commission II/5, WG II/10

KEY WORDS: Image Restoration, Regularization, Total Variation, Parameter Estimation, Residual Similarity, Convex Minimization

ABSTRACT:

Image restoration with regularization models is very popular in the image processing literature. Total variation (TV) is one of the important edge preserving regularization models used, however, to obtain optimal restoration results the regularization parameter needs to be set appropriately. We propose here a new parameter estimation approach for total variation based image restoration. By utilizing known noise levels we compute the regularization parameter by reducing the similarity between residual and noise variances. We use the split Bregman algorithm for the total variation along with this automatic parameter estimation step to obtain a very fast restoration scheme. Experimental results indicate the proposed parameter estimation obtained better denoised images and videos in terms of PSNR and SSIM measures and the computational overload is less compared with other approaches.

1. INTRODUCTION

Total variation (TV) regularization is introduced by Rudin et al. (Rudin et al., 1992) for obtaining edge preserving restorations of noisy images. Due to its edge preserving property the TV model is used widely in image processing and other related areas (Prasath et al., 2015). If we let \( f : \Omega \subset \mathbb{R}^2 \rightarrow [0, 255] \) be the (noisy) input image, \([I] = m \times n \) is the image domain and size. Solving the following unconstrained minimization provides an estimate of the latent image \( f = u + \epsilon \) with additive Gaussian noise \( \epsilon \) of standard deviation \( \sigma \) (assumed known),

\[
\min_u \mathcal{E}_\lambda(u) = \lambda TV(u) + \sum_{1 \leq i,j \leq M,N} (u_{ij} - f_{ij})^2,
\]

where \( TV(u) = \sum_{1 \leq i,j \leq m,n} |(\nabla u)_{ij}| \) is the discrete total variation with \((\nabla u)_{ij} = (u_{i+1,j} - u_{ij}, u_{i,j+1} - u_{ij})^T\), with zero boundary conditions. The regularization parameter \( \lambda > 0 \) is crucial in obtaining meaningful results and traditionally it is left as a tunable parameter (Prasath et al., 2017, Prasath et al., 2018). Larger \( \lambda \) values result in over-smoothing whereas smaller \( \lambda \) value parameter results in noise being kept, see Figure 1. As can be seen from the residual images (Fig. 1(middle row)) more structures are removed as we increase the \( \lambda \) value. Thus, a systematic approach in selecting this crucial regularization parameter is paramount in obtaining accurate denoising results. There exists various methods for choosing the regularization parameter such as the Lagrange multiplier (Rudin et al., 1992), discrepancy principle, L-curve, cross-validation (Hansen, 2010) and generalized cross validation (Reeves, Mersereau, 1990) etc. Fixing a parameter with these standard parameter estimation methods can still lead to poor results, since the regularization needs to be reduced in the iterative implementation. Recently data adaptive methods (Fu, Zhang, 2010, Prasath, Singh, 2010, Wang et al., 2011, Prasath, 2011, Prasath, Moreno, 2018) are found to provide better solutions, though with higher computational overhead.

If we assume that the noise level is known (or an upper bound), then we can utilize it to tune \( \lambda \) dynamically through the iterations. This means that we solve (1) for both the image and \( \lambda \), which is a bi-objective optimization (Weiss et al., 2009, Beck, Teboulle, 2009). In this work, our main aim is to use a similarity criteria based parameter estimation for TV minimization model with a fast implementation. Among a wide variety of iterative methods available to solve the TV minimization (Vogel, Oman, 1998, Dobson, Vogel, 1997, Blomgren et al., 1997, Chambolle, 2004, Osher et al., 2005), recently the split Bregman (Goldstein, Osher, 2009) is proven to be very fast and used widely in the literature. By using the fact that in TV based denoising with an iterative algorithm the similarity between the residual and noise variance (SIMRES) should approach zero, we devise an algorithm that can help select the regularization parameter \( \lambda \) effectively. We test the SIMRES-TV on a variety noisy images and compare with other regularization parameter selection models (Rudin et al., 1992, Hansen, 2010, Reeves, Mersereau, 1990, Prasath, Singh, 2010). Our experimental results indicate that SIMRES-TV can obtain better results with comparable or exceeding image quality in denoising natural images. Further, the proposed SIMRES approach, apart from with TV, can also...
Figure 1. Top row: TV regularization (1) based image restoration results for different constant \( \lambda \) values. Middle row: Noise amount (shown in (a) with gray values reversed for better visualization), and corresponding residual images \( |u - f| \). Bottom row: Structural similarity (SSIM) maps between original image and resultant images. Larger \( \lambda \) blurs edges and converges to a piecewise constant image.

be augmented to any convex regularization functions with convergence guarantees.

Our paper is organized as follows. Section 2 introduces the SIMRES-TV algorithm. Section 3 provides the experimental details and comparison of denoising various natural images. Finally, Section 4 concludes the paper.

2. PARAMETER ESTIMATION WITH SIMRES CRITERIA

We first note that the similarity between residual and noise variances (SIMRES) can be quantified with the following equation:

\[
R(u) = \frac{[(u - f)^2 / MN] - \sigma^2}{\sigma^2} \quad (2)
\]

Ideally, in image denoising, we require that \( R(u) \to 0 \), however in typical iterative optimization schemes, it can take a large number of iterations to achieve such requirement, and usually constraining the residue closer to zero, \( R(u) < Th \) with \( Th \) small, would suffice to obtain meaningful restorations. Imposing such a constraint on \( R(u) \) along with minimizing a regularization such as the TV (1) can be undertaken and that leads to selecting appropriate regularization parameter \( \lambda \). We thus use a decision based on the smaller SIMRES criterion for determining the \( \lambda \) parameter and use it to minimize the TV objective functional (1). To motivate this, we present an example of denoising a natural image corrupted with Gaussian noise in Figure 1 with different \( \lambda \) values based TV regularization. As can be seen, setting \( \lambda \) high removes finer details and obtains piecewise constant results. In contrast, using smaller \( \lambda \) values fails to obtain cleaner results. Instead of hand-tuning the parameter, in this work, we embed decreasing \( \lambda \) values which were tuned by the relative residual term into the minimization of (1). The overall algorithm is as follows:

1. Initialise \( \lambda^1 \), \( Th \), and \( u^0 = f \).

   (a) We use mean filtered (3 × 3 averaging kernel) image \( \tilde{f} \) of input \( f \) to compute the initial \( \lambda^1 = \xi^0 \frac{(f_{ij} - \tilde{f}_{ij})^2}{TV(f)} \) with initial value \( \xi^0 = 0.5 \).

   (b) Compute a minimiser of Eqn. (1),

\[
\arg\min_v E_{\lambda^1}(v). \quad (3)
\]

2. For \( k = 2, \ldots \) do:

   (a) Reduce the parameter value using the following sequence,

\[
\lambda^k = \xi^{k-1} \frac{\sum_{ij}(u^k_{ij} - f_{ij})^2}{TV(u^k_{ij})} \quad (4)
\]

   where \( \xi^{k-1} > 0 \) is a parameter.

   (b) Compute a minimiser of Eqn. (1),
The scaling parameter is chosen to be decreasing, i.e., $\xi^k = \xi^{k-1} / \gamma$ and this guarantees the convergence of our algorithm to a minimal solution of (1). Note that we only compute an approximate solution (local-minima) of the energy functional (1) at these steps. Since the TV regularization based minimization is convex, we are guaranteed to find a minima and the solution corresponds to an optimal denoising result in our case. The piecewise constant regions can be observed. IG (Prasath, Singh, 2010) along with TV minimization.

3. EXPERIMENTAL RESULTS AND COMPARISONS

All the images and variables are normalized to $[0, 1]$ range, and noise level $\sigma^2 = 0.01$. The threshold $Th = 0.1$, scaling parameter $\gamma = 10$ and tolerance $Tol = 10^{-12}$ are fixed. Tolerance based stopping criteria is used for other schemes reported here. We compare with the Lagrange multiplier (LM) (Rudin et al., 1992), inverse gradient (IG) (Prasath, Singh, 2010, Prasath, Moreno, 2018, Thanh et al., 2020), generalized cross validation (GCV) (Reeves, Mersereau, 1990) along with TV minimization.

Table 2 shows restoration of Elaine grayscale image for different parameter estimation methods. We show in 2(a) top row the original noise-free image, and bottom row the amount of noise that need to be removed. Residual images clearly show that SIMRES based restoration is better in terms of edge preservation and does not remove edge pixels as observed in the other results. In LM (Rudin et al., 1992) based result we see uniform blurring, whereas in GCV (Reeves, Mersereau, 1990) piecewise constant regions can be observed. IG (Prasath, Singh, 2010) provided better results than LM, and GCV but removed some edges as can be seen on the corresponding residue image. In contrast, the proposed SIMRES-TV model obtained better result without removing edges, compare the random noise in (a) with the residue image in (e).

For quantitative measurements we utilize the peak signal to noise ratio - PSNR (in decibels, dB) and the structural similarity - SSIM (range [0,1]) image quality metrics. Table 1 shows the PSNR, mean SSIM values and computation time (in seconds) corresponding to restorations shown in Figure 2. Apart from achieving better quality values, the SIMRES-TV based implementation is faster as well as computationally efficient. In terms of edge preservation, SSIM value indicate that the structural details are kept well by the SIMRES-TV with more than 6 dB increase in PSNR values over the nearest model (Prasath, Singh, 2010).
Figure 3 shows the energy value ($E_2$) of size $200 \times 200 \times 3$. Figure 4 shows a comparison of restoration results of the methods based on the color-TV (Blomgren, Chan, 1998; Bresson, Chan, 2008) for Gaussian noise of variances $\sigma_n = 20$. We can see that our SIMRES approach consistently obtains the highest SSIM values among different methods indicating that edges are preserved well across various natural images.

Finally, we provide an example restoration of our SIMRES-TV using a color RGB image *Tulip* of size $200 \times 200 \times 3$. Figure 3 shows the energy value ($E_2$) of size $200 \times 200 \times 3$. Figure 4 shows a comparison of restoration results of the methods based on the color-TV (Blomgren, Chan, 1998; Bresson, Chan, 2008) for Gaussian noise of variances $\sigma_n = 30, 50$ added in each channel. Compared to the traditional color-TV model with either lower $\lambda = 0.1$ (noise retention) or higher $\lambda = 0.3$ (noise suppression) or higher order TV models (Thanh et al., 2020) are also an interesting direction of research.

Next, we show in Table 2 SSIM values for various methods compared with our proposed SIMRES-TV in different standard test images taken from the USC-SIPI Miscellaneous dataset. The input images are corrupted with Gaussian noise of variance $\sigma_n = 20$. We can see that our SIMRES approach consistently obtains the highest SSIM values among different methods indicating that edges are preserved well across various natural images.

Table 3. Comparison of different quality and error metrics for the *Tulip* color image with TV regularization for two different noise levels. Best results are in **boldface**.

<table>
<thead>
<tr>
<th>Images</th>
<th>LM</th>
<th>GCV</th>
<th>IG</th>
<th>Our SIMRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Couple</td>
<td>0.7963</td>
<td>0.7298</td>
<td>0.8224</td>
<td></td>
</tr>
<tr>
<td>F-16</td>
<td>0.8128</td>
<td>0.8178</td>
<td>0.8441</td>
<td></td>
</tr>
<tr>
<td>Girl1</td>
<td>0.7930</td>
<td>0.8291</td>
<td>0.8681</td>
<td></td>
</tr>
<tr>
<td>Girl2</td>
<td>0.8934</td>
<td>0.822</td>
<td>0.8911</td>
<td></td>
</tr>
<tr>
<td>Girl3</td>
<td>0.7825</td>
<td>0.8006</td>
<td>0.8178</td>
<td></td>
</tr>
<tr>
<td>House</td>
<td>0.7299</td>
<td>0.8128</td>
<td>0.8513</td>
<td></td>
</tr>
<tr>
<td>IPI</td>
<td>0.8841</td>
<td>0.8745</td>
<td>0.9204</td>
<td></td>
</tr>
<tr>
<td>IPIC</td>
<td>0.8824</td>
<td>0.8925</td>
<td>0.9238</td>
<td></td>
</tr>
<tr>
<td>Tree</td>
<td>0.7385</td>
<td>0.704</td>
<td>0.8129</td>
<td></td>
</tr>
<tr>
<td>Baboon</td>
<td>0.5687</td>
<td>0.4701</td>
<td>0.6032</td>
<td></td>
</tr>
<tr>
<td>Barbara</td>
<td>0.6892</td>
<td>0.7131</td>
<td>0.8671</td>
<td></td>
</tr>
<tr>
<td>Boat</td>
<td>0.6973</td>
<td>0.6078</td>
<td>0.7839</td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td>0.7891</td>
<td>0.677</td>
<td>0.8431</td>
<td></td>
</tr>
<tr>
<td>Lena</td>
<td>0.7573</td>
<td>0.8434</td>
<td>0.8997</td>
<td></td>
</tr>
<tr>
<td>Peppers</td>
<td>0.8942</td>
<td>0.8513</td>
<td>0.9105</td>
<td></td>
</tr>
<tr>
<td>Splash</td>
<td>0.8967</td>
<td>0.8876</td>
<td>0.9271</td>
<td></td>
</tr>
<tr>
<td>Tiffany</td>
<td>0.7596</td>
<td>0.7248</td>
<td>0.8478</td>
<td></td>
</tr>
</tbody>
</table>

In this work, we studied a noise and residual similarity measure based parameter estimation for TV regularization scheme in image restoration. The proposed SIMRES approach for estimating the regularization parameter automatically as part of the TV minimization is general in the sense that it can be used along with other (possibly) non-convex minimizations. It will also be interesting to add an independent noise estimation step which will make the proposed method completely automatic for various image processing tasks. Also, combining with other regularization models that utilize adaptive regularization (Prasath, Thanh, 2019) or higher order TV models (Thanh et al., 2020) are also an interesting direction of research.

ACKNOWLEDGEMENTS

This work was funded by the University of Economics Ho Chi Minh City, Vietnam.

REFERENCES

Beck, A., Teboulle, M., 2009. Fast Gradient-Based Algorithms for Constrained Total Variation Image Denoising and
Figure 4. Color image restoration with TV regularization with constant regularization parameter and SIMRES approach. (a) Original and (b) noisy images (with Gaussian noise variances $\sigma_n = 30$ and $\sigma_n = 50$). Restoration results with TV regularization parameter (c-d) $\lambda = 0.1$, (e-f) $\lambda = 1$, and (g-h) our SIMRES-TV. In each of the restoration results, we show restored images on the left and corresponding residual images on the right.


