MULTIPLE REFLECTION EFFECTS IN NONLINEAR MIXTURE MODEL FOR HYPERSPECTRAL IMAGE ANALYSIS

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ABSTRACT:

Hyperspectral spectrometers can record electromagnetic energy with hundreds or thousands of spectral channels. With such high spectral resolution, the spectral information has better capability for material identification. Because of the spatial resolution, one pixel in hyperspectral images usually covers several meters, and it may contain more than one material. Therefore, the mixture model must be considered. Linear mixture model (LMM) has been widely used for remote sensing target classifications, because of its simplicity and yields reasonable results for smooth surfaces. For rough surfaces, the physical interactions of the light scattered between multiple materials in the scene must be considered. Recently, Generalized Bilinear Model (GBM) is proposed and it includes the double reflection between different materials into a nonlinear model, but it ignores the interactions within the same material. In this study, we propose a modified version of GBM to further consider this effect in our model, called Modified Generalized Bilinear Model (MGBM).

1. INTRODUCTION

Hyperspectral spectrometers can record electromagnetic energy with hundreds or thousands of spectral channels. With such high spectral resolution, the spectral information has better capability for material identification. Because of the instantaneous field of view (IFOV) of one spectrum usually cover several meters, it may contain more than one material. Therefore, the mixture model must be considered. Linear mixture model (LMM) has been widely used for remote sensing target classifications (Keshava, 2002 and Manolakis, 2001), because of its simplicity and yields reasonable results for smooth surfaces. But it can be inappropriate for rough surfaces where the physical interactions of the light scattered between multiple materials in the scene. Recently, Generalized Bilinear Model (GBM) is proposed and it includes the interaction between different materials which results a nonlinear model with second order interactions between two materials. For rough surface, multiple reflections must be considered. Let \( y \) be an \( L \times 1 \) column vector and denote the spectral signature of one spectrum, where \( L \) is the number of spectral bands. And \( M \) is an \( L \times p \) endmembers matrix denoted by \( \{ \mathbf{m}_1, \ldots, \mathbf{m}_R \} \), where \( R \) is number of materials and \( \mathbf{m}_j \) is an \( L \times 1 \) column vector representing the \( j \)-th endmember signature resident in \( y \). Let \( \mathbf{a} = [a_1, a_2, \ldots, a_R]^T \) be a \( p \times 1 \) abundance column vector associated with \( y \) and \( \mathbf{n} \) is an additive white noise. Then LMM can be represented by following equation.

\[
\mathbf{y} = M \mathbf{a} + \mathbf{n}
\]

(1)

For rough surface, multiple reflections must be considered. Generalized Bilinear Model considers second-order interactions between \( i \)-th and \( j \)-th endmembers (for \( i, j = 1, \ldots, R \) and \( i \neq j \)) and the observed mixed pixel \( \hat{y} \) can be written as

\[
\mathbf{y} = \mathbf{y}^{\mathbf{a}} + \mathbf{y}^{\mathbf{n}}
\]

\[
\mathbf{y}^{\mathbf{a}} = \mathbf{M} \mathbf{a}
\]

\[
\mathbf{y}^{\mathbf{n}} = \mathbf{M} \mathbf{a} \mathbf{a}^T \mathbf{n}
\]

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\[
\hat{y} = M\hat{a} + \sum_{i=1}^{R-1} \sum_{j=1}^{R} \gamma_{i,j} \alpha_i \alpha_j \hat{m}_i \otimes \hat{m}_j + \hat{n}
\]  

(2)

where \( \otimes \) denotes that Hadamard product operation and the \( \gamma_{i,j} \) is the nonlinear mixing coefficient that controls the interaction between endmembers \( i \) and \( j \).

3. MODIFIED GENERALIZED BILINEAR MODEL AND FAN-FCLS ALGORITHM

The MGBM is a nonlinear model that considers the second order interactions within the same material. The corresponding mixed pixel \( \hat{y} \) is then expressed as

\[
\hat{y} = M\hat{a} + \sum_{i=1}^{R-1} \sum_{j=1}^{R} \gamma_{i,j} \alpha_i \alpha_j \hat{m}_i \otimes \hat{m}_j + \hat{n}
\]  

(3)

Because the MGBM is a nonlinear problem, it is difficult to solve. Fan-FCLS algorithm is proposed to solve the optimization problem (Fan, 2009). Estimating the MGBM parameter vector can be formulated as following optimization problem

\[
\hat{\theta} = \arg \min_{\theta} \left\| \hat{y} - \bar{\mu}(\hat{\theta}) \right\|^2
\]  

(4)

where

\[
\bar{\mu}(\hat{\theta}) = M\hat{a} + \sum_{i=1}^{R} \sum_{j=1}^{R} \gamma_{i,j} \alpha_i \alpha_j \hat{m}_i \otimes \hat{m}_j + \hat{n}
\]  

(5)

Since it is a nonlinear problem, Taylor series expansion is applied to solve it. At iteration \( t \) of the algorithm with point estimation

\[
\hat{\theta}^t = (\alpha_1^t, \ldots, \alpha_R^t, \gamma_{1,1}^t, \ldots, \gamma_{R,R}^t)
\]  

(6)

the linearization yields

\[
\bar{\mu}(\hat{\theta}) \approx \bar{\mu}(\hat{\theta}^t) + \frac{\partial \bar{\mu}}{\partial \hat{\theta}}|_{\hat{\theta} = \hat{\theta}^t} (\hat{\theta} - \hat{\theta}^t)
\]  

(7)

The iterative updating rule can be written as follow

\[
\hat{\theta}^{t+1} = \arg \min_{\theta} \left\| \hat{y} - \bar{\mu}(\hat{\theta}^t) + g^t \hat{\theta} \right\|^2
\]  

(8)

and

\[
\hat{h}^t = \hat{y} - \bar{\mu}(\hat{\theta}^t) + g^t \hat{\theta}^t
\]  

(9)

Secondly, the MGBM are subject to sum-to-one and nonnegative constraints. The optimization problem is applying the fully constrained least square (FCLS) algorithm. The constraints can be handled by using the extended matrix

\[
G' = \begin{pmatrix}
\delta g_{L,S} & 0_{L,S} \\
0_{S,R} & I_{S,S} \\
I_{R,1}^T & 0_{S,1}^T & 0_{S,1}^T
\end{pmatrix}
\]  

(10)

and

\[
H' = \begin{pmatrix}
\delta \hat{h}'^T \\
1_S \\
1
\end{pmatrix}
\]  

(11)

The stopping criterion is met when the difference between the estimations of next iteration is less than a preset threshold.

4. EXPERIMENTAL RESULT

The data set for experiment was taken from ROSIS sensor at the University of Pavia in 2001. There are 103 spectral bands after removing water absorption channels. A subset of 35x40 pixels is selected for experiment as shown in Fig 1. There are three endmembers in this image: Road, Tree and Grass. The surfaces of these three endmembers have different roughness. Road is smooth and expect to have low multiple reflection and the tree has the roughest surface which expect to have high multiple reflection.

Figures 2, 3 and 4 are the abundance fraction maps estimated by LMM, GBM and MGBM respectively. The result shows the abundance estimation of Road, Tree and Grass are similar among these three models. Figure 3 also shows the second-order reflection between different materials for GBM, and this phenomenon highly appears at the boundary of two materials. MGBM further considers the second-order reflection within the same material as shown in Figure 4. The color bars beside the fraction maps indicate the higher possibility of multiple reflections for Tree than Road and Grass, which confirms our expectation.
5. CONCLUSION

Mixture model has been widely implemented for remote sensing for several decades. The Linear Mixture Model neglects multiple reflections and assumes linear combination which is suitable for smooth surface. However, the interaction between materials has to be considered for rough surface. The proposed Modified Generalized Bilinear Model is developed from Generalized Bilinear Model by further include second order reflection within the same material, and the preliminary results support our assumption. The experiment with in situ measurement under controlled environment will be conducted and analyzed in the near future.

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