HYPERSPECTRAL IMAGE KERNEL SPARSE SUBSPACE CLUSTERING WITH SPATIAL MAX POOLING OPERATION

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ABSTRACT:

In this paper, we present a kernel sparse subspace clustering with spatial max pooling operation (KSSC-SMP) algorithm for hyperspectral remote sensing imagery. Firstly, the feature points are mapped from the original space into a higher dimensional space with a kernel strategy. In particular, the sparse subspace clustering (SSC) model is extended to nonlinear manifolds, which can better explore the complex nonlinear structure of hyperspectral images (HSIs) and obtain a much more accurate representation coefficient matrix. Secondly, through the spatial max pooling operation, the spatial contextual information is integrated to obtain a smoother clustering result. Through experiments, it is verified that the KSSC-SMP algorithm is a competitive clustering method for HSIs and outperforms the state-of-the-art clustering methods.

1. INTRODUCTION

Hyperspectral sensors can acquire nearly continuous spectral bands with hundreds of channels to capture the diagnostic information of land-cover materials(Zhang, et. al., 2014a), which opens up new possibilities for remote sensing applications, such as mineral exploration, fine agriculture, disaster monitoring, and so on (Landgrebe, 2002, Zhang, et. al., 2014b) . As an unsupervised information extraction technique, clustering is a basic tool of hyperspectral image (HSI) applications. However, due to the complex nonlinear structure and large spectral variability, clustering HSIs is still a very challenging task.
The traditional HSI clustering methods, such as *k*-means (Lloyd, 1982) and fuzzy c-means (FCM) (Bezdek, 2013), attempt to segment pixels using only spectral measurements. Unfortunately, such methods often fail to achieve satisfactory clustering results because of the limited discriminative capability. Therefore, in recent years, researchers have begun to develop spectral-spatial clustering methods which consider spectral measurements together with spatial information to improve the clustering performance, such as FCM_S1 (Chen, and Zhang, 2004), and *k*-means_S (Luo, et al., 2003). However, these methods still have limited clustering performance due to large spectral variability of HSIs.

The sparse subspace clustering (SSC) algorithm was recently proposed (Elhamifar, and Vidar, 2013), and has achieved great success in the face recognition and motion segmentation fields. Based on the assumption that pixels with different spectra from one land cover class lie in the same subspace, the SSC algorithm shows great potential in HSI clustering. Based on the subspace model, the spectral variability problem can be effectively relieved (Zhang, et. al., 2016). Recently, a spectral-spatial sparse subspace clustering algorithm was proposed to make full use of the spectral-spatial information of HSIs (Zhang, et. al., 2016), which significantly improves the clustering performance. However, the SSC model is based on the linear subspace model, while HSIs are generally considered to be linearly inseparable. Therefore, these linear models cannot cope well with the inherently nonlinear structure of the HSIs.

In recent years, a number of approaches have been proposed to deal with the linearly inseparable obstacle in the classification field. Among them, the kernel strategy is one of the most commonly used and effective methods. This approach maps the HSI from the original feature space to a higher kernel feature space to make the problem linearly separable, which has been successfully used in the kernel-based SVM classifier (Mercier, and Lennon, 2003) . However, to the best of our knowledge, in the HSI clustering domain, few clustering methods have been proposed to deal with the nonlinear structure of the HSIs up to date.

In view of this, in this paper, a novel kernel sparse subspace clustering algorithm with spatial max pooling operation (KSSC-SMP) for hyperspectral remote sensing imagery was proposed, which simultaneously explores the nonlinear structure and the inherent spectral-spatial attributes of HSIs. Firstly, we map the feature points from the original feature space to a higher-dimensional space with the kernel strategy to make the feature points linearly inseparable. And then, in order to fully exploit the spectral-spatial discrimination information of HSIs and the potential of the SSC model, the spatial max pooling operation is introduced to incorporate the spatial information to improve the clustering performance and guarantee spatial homogeneity of the clustering result.

2. SPARSE SUBSPACE CLUSTERING

In the SSC model, for an HSI with the size of $M \times N \times p$, all the pixels can be seen as selected from a union of l affine subspaces $S_1 \cup S_2 \cup \cdots S_l$ of dimensions $\{d_i\}_{i=1}^{l}$ in the full space $\mathbb{R}^p$ with $d_1 + d_2 + \cdots d_l = p$, where M denotes the height of the image, N represents the width of the image, and p is the number of band channels. By treating each pixel as a column vector, the HSI cube can be transformed into a 2-D matrix $\mathbf{Y} = [y_1, y_2, \cdots, y_{MN}] \in \mathbb{R}^{p \times MN}$. Then, with this 2-D-hyperspectral matrix itself being used as the representation dictionary, the sparse coefficient matrix can be obtained by solving the following optimization problem:

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$$\min_{\mathbf{C}} \|\mathbf{C}\|_0 \quad s.t \quad \mathbf{Y} = \mathbf{YC} + \mathbf{N},\ diag(\mathbf{C}) = \mathbf{0},\ \mathbf{C}^T\mathbf{1} = \mathbf{1} \qquad (1)$$

where $\mathbf{C} \in \mathbb{R}^{MN \times MN}$ represents the representation coefficient matrix, $\mathbf{N} \in \mathbb{R}^{p \times MN}$ denotes the representation error matrix, and $\mathbf{1} \in \mathbb{R}^{MN}$ is a vector whose elements are all ones. The condition $diag(\mathbf{C}) = 0$ is utilized to eliminate the trivial solution of each pixel being represented as a linear combination of itself (Elhamifar, and Vidar, 2013). The condition $\mathbf{C}^T\mathbf{1} = \mathbf{1}$ means that it adopts the affine subspace model, which is a special linear subspace model. As the ℓ_0 - norm optimization problem is NP-hard, the relaxed tractable ℓ_1 -norm is usually adopted:

$$\min_{\mathbf{C}} \|\mathbf{C}\|_1 \quad s.t \quad \mathbf{Y} = \mathbf{YC} + \mathbf{N},\ diag(\mathbf{C}) = \mathbf{0},\ \mathbf{C}^T\mathbf{1} = \mathbf{1} \qquad (2)$$

The optimization problem in (2) can be effectively solved by the alternating direction method of multipliers (ADMM) algorithm (Mota, et. al., 2013). We then construct the similarity graph with the obtained coefficient matrix in the symmetric form to strengthen connectivity of the graph (Elhamifar and Vidar, 2013).

$$\mathbf{W} = |\mathbf{C}| + |\mathbf{C}|^T \qquad (3)$$

The spectral clustering algorithm is then applied to the similarity graph to obtain the final clustering result (Ng, et. al., 2002) .

3. THE KERNEL SPARSE SUBSPACE CLUSTERING ALGORITHM WITH SPATIAL MAX POOLING OPERATION

In this section, we attempt to fully exploit the inherently nonlinear structure and the spectral-spatial properties of HSIs to achieve more accurate clustering results. For the former purpose, we adopt the kernel strategy to make the linearly inseparable feature points become linearly separable. And for the latter one, we incorporate the spatial max pooling operation into the clustering scheme after obtaining the coefficient matrix to effectively exploit the spatial information.

3.1 The Kernel Sparse Subspace Clustering Algorithm

The kernel sparse subspace clustering algorithm (KSSC) extends SSC to nonlinear manifolds by using the kernel strategy to map the feature points from the original space to a higher kernel space, in order to make them linearly separable (Patel, and Vidal, 2014). Then, the kernel sparse representation coefficient matrix can be acquired by solving the following optimization problem, with the kernelized data matrix being used as the self-representation dictionary.

$$\begin{aligned} &\min_{\mathbf{C}} \|\mathbf{C}\|_1 + \lambda \|\phi(\mathbf{Y}) - \phi(\mathbf{Y})\mathbf{C}\|_F^2 \\ &s.t\ diag(\mathbf{C}) = \mathbf{0},\ \mathbf{C}^T\mathbf{1} = \mathbf{1} \end{aligned} \qquad (4)$$

where λ is a tradeoff between the data fidelity term and the sparse term; and $\phi(\cdot): \mathbb{R}^p \rightarrow \mathrm{H}$ is a mapping function from the input space to the reproducing kernel Hilbert space H (Patel, and Vidal, 2014). The kernel matrix is usually defined as $\kappa(\mathbf{y}_i, \mathbf{y}_j) = \langle \phi(\mathbf{y}_i), \phi(\mathbf{y}_j) \rangle = \phi(\mathbf{y}_i)^T \phi(\mathbf{y}_j)$, where $\kappa(\bullet): \mathbb{R}^p \times \mathbb{R}^p \rightarrow \mathbb{R}$ represents the kernel function, which measures the similarity of two arguments denoting a pair of pixels. The commonly used kernels include the radial basis function (RBF) kernel $\kappa(\mathbf{y}_i, \mathbf{y}_j) = \exp\left(-\delta \|\mathbf{y}_i - \mathbf{y}_j\|^2\right)$ and the polynomial kernel $\kappa(\mathbf{y}_i, \mathbf{y}_j) = \left\langle \langle \mathbf{y}_i, \mathbf{y}_j \rangle + a \right\rangle^b$, where δ, a, and b are the parameters of the kernel functions. In this paper, the RBF kernel is adopted. As the feature space of the RBF kernel has an infinite number of dimensions, the value of the RBF kernel decreases with distance and ranges between [0, 1], which can be readily interpreted as a similarity measure.

3.2 Incorporating Spatial Information with the Spatially Max Pooling Operation

The spatial pooling operation, as an effective way to extract local statistical information, has been used in various fields, including remote sensing analysis (Yuan, and Tang, 2014). Through sparse representation procedure, the coefficients can reveal the underlying cluster structure and can be used as the classification features. However, the single sparse vector of each pixel can only provide limited discriminative information as it only contains spectral measurements. In order to increase the discriminative ability for object recognition, a reasonable approach is to incorporate the spatial information to assist the spectral analysis. After obtaining the sparse coefficient matrix, a natural idea is to merge the sparse coefficients of spatially adjacent pixels in a local window to generate a new feature vector with the spatial pooling operation (Yuan, and Tang, 2014). Considering the characteristics and mechanism of SSC, we focus on the larger elements of each coefficient vector as they have a larger probability of being from the same subspace. Therefore, the spatial max pooling operation is adopted to merge these sparse representation vectors into a pooling vector, which suppresses the low elements and preserves the large ones. By this way, the spatial contextual information is effectively incorporated into the new coefficients to generate the spectral-spatial features. These pooling coefficients are then used as the features for the subsequent clustering.

3.3 The KSSC-SMP Algorithm

In order to further improve the clustering performance, we combine the two schemes into a unified framework to obtain the KSSC-SMP algorithm, which can simultaneously deal with the complex nonlinear structure and utilize the spectral-spatial attributes of HSIs.

After spatially max pooling the sparse representation coefficient matrix in (4), the similarity graph is constructed through (3), similar to SSC. And the final clustering result is achieved by applying spectral clustering to it.
The proposed KSSC-SMP algorithm can be summarized as algorithm 1.

Algorithm 1: The kernel sparse subspace clustering algorithm with spatial max pooling operation (KSSC-SMP) for hyperspectral remote sensing imagery.

Input:
1) A 2-D matrix of the HSI containing a set of points $\{y_i\}_{i=1}^{MN}$, in a union of l affine subspaces $\{C_i\}_{i=1}^{l}$;
2) Parameters, including the cluster number l, the regulation parameter λ, the kernel parameter δ, and the window size n of the spatial max pooling operation.

Main algorithm:
1) Construct the kernel sparse representation optimization model (4) and solve it to obtain the kernel sparse representation coefficient matrix $\mathbf{C}$ using ADMM;
2) Conduct the spatial max pooling operation on $\mathbf{C}$ to obtain the pooling coefficient matrix;
3) Construct the similarity graph with the pooling coefficient matrix;
4) Apply spectral clustering to the similarity graph to obtain the final clustering results;

Output:
A 2-D matrix which records the labels of the clustering result of the HSI.

4. EXPERIMENTAL RESULTS AND DISCUSSION

4.1 Experimental Setting

The proposed KSSC-SMP algorithm was tested on two widely used hyperspectral datasets: the Indian Pines image and the University of Pavia image. Several different clustering methods were implemented for comparisons: clustering by fast search and find of density peaks (CFSFDP) (Rodriguez, and Laio, 2014), SSC (Elhamifar, and Vidar, 2013), FCM_S1 (Chen, and Zhang, 2004) and S^4C (Zhang, et. al., 2016). Two simplified versions of the proposed KSSC-SMP method, Kernel SSC (KSSC) and SSC with spatial max pooling (SSC-SMP) are also included. The number of clusters was set as equal to the number of classes in the ground truth. For CFSFDP, we manually selected the clusters in the decision graph generated by the algorithm to obtain the final clustering result. The other parameters of each clustering method were manually tuned to the optimum. Both visual clustering maps and quantitative evaluations of the precision (producer's accuracy, user's accuracy, overall accuracy (OA), and kappa coefficient) are given to thoroughly evaluate the clustering performance of each method.

The AVIRIS Indian Pines image is used to conduct the first real data experiment. The image is of size $145\times145\times200$. The same portion of Indian Pines data with the size of $85\times70\times200$ in (Zhang, et. al., 2016) is utilized to test the performance of the proposed algorithm. This typical test area contains four main kinds of ground materials, which is very challenging for clustering because the ground materials in this area have very similar spectra. The parameters of KSSC-SMP are set as follows: the number of clusters was set to 4 and the size of the window was set to $n=3$, and $\lambda=7\times10^{-7}$.

The ROSIS Pavia University image is used to conduct the second real data experiment. The data size is 610 lines by 340 samples, with 103 bands. As before, we use a typical subset for tested data with size of $85\times70\times103$, which includes four main kinds of materials. The parameters of KSSC-SMP are set as follows: the number of clusters was set to 4 and the size of the window was set to $n=3$, and $\lambda=1\times10^{-5}$.

4.2 Experimental Results

The clustering result of each clustering algorithm on Indian Pines image is shown in Fig.1 with the corresponding quantitative evaluations shown in Table I. In the table, the optimal value of each row is shown in bold, and the second-best results are underlined. From Table I, it can be clearly observed that the proposed KSSC-SMP algorithm outperforms the others. The performance of the CFSFDP and SSC algorithms are unsatisfactory and contain a lot of salt-and-pepper noise and misclassifications. Compared with SSC, KSSC improves the clustering performance to a large degree by exploring the nonlinear structure of the HSIs with the kernel strategy. In the KSSC result, most of the classes are successfully separated. For the soybeans-minimum-till class, the misclassification is significantly reduced and the recognition rate is improved from 58.26% to 78.59%. The SSC-SMP method performs better than SSC because of the incorporation of the spatial information with the spatial max pooling operation, and the noise in the grass, soybeans-no-till, and soybeans-minimum-till classes is well smoothed. By incorporating the spatial information, the FCM_S1 and S^4C algorithms also obtain smoother clustering results. Finally, by integrating the kernel technique and the spatial max pooling operation into one framework, the proposed KSSC-SMP algorithm performs better than KSSC and SSC-SMP to achieve the best clustering result, both visually and quantitatively, with the best OA of 80.37% and a kappa coefficient of 0.7134. A 13% improvement in OA is achieved compared with respect to SSC, which proves the effectiveness and superiority of the proposed method.

Fig.2 shows the clustering result of Pavia University image and Table II presents the clustering accuracy. The clustering results further suggested the superiority of the proposed KSSC-SMP method. CFSFDP and SSC achieve inferior clustering results with a low accuracy. KSSC performs much better than SSC and obtains a much higher accuracy, which further proves the effectiveness of the kernel strategy and the superiority of the nonlinear method. Meanwhile, SSC-SMP performs better than SSC and obtains a much smoother clustering result. FCM_S1 and S^4C also obtain smoother clustering maps with a higher accuracy. The proposed KSSC-SMP algorithm once achieves the best clustering accuracy, which can be easily seen visually and via the quantitative evaluations of the precision in Table II.

Method	Class	Corn-no-till	Grass	Soybeans-no-till	Soy-minimum-till	OA	Kappa
CFSFDP	Producer's accuracy (%)	31.84	**100**	**76.09**	55.25	60.81	0.4589
FCM_S1		63.93	95.07	44.95	62.09	65.52	0.5145
SSC		**67.16**	92.19	66.12	58.26	67.25	0.5488
S^4C		61.00	**100**	65.30	65.28	70.08	0.5825
KSSC		61.99	99.86	52.32	78.59	73.95	62.43
SSC-SMP		52.14	**100**	70.49	90.38	79.91	0.7064
KSSC-SMP		49.45	**100**	75.41	**90.96**	**80.37**	**0.7134**

Table 1. Quantitative Evaluation of the Different clustering Algorithms with the Indian Pines Image.

Method	Class	Metal sheet	Trees	Shadows	Bare soil	OA	Kappa
CFSFDP	Producer's accuracy (%)	**99.92**	100	100	16.70	59.41	0.5950
FCM_S1		**99.92**	100	100	31.27	66.50	0.5416
SSC		99.17	100	0	37.53	61.84	0.4223
S^4C		99.00	100	100	46.65	73.59	0.6208
KSSC		99.75	100	100	33.11	67.32	0.5506
SSC-SMP		99.25	100	100	45.70	73.24	0.6165
KSSC-SMP		**99.92**	100	100	**47.98**	**74.63**	**0.6336**

Table 2. Quantitative Evaluation of the Different clustering Algorithms with the Pavia University Image.

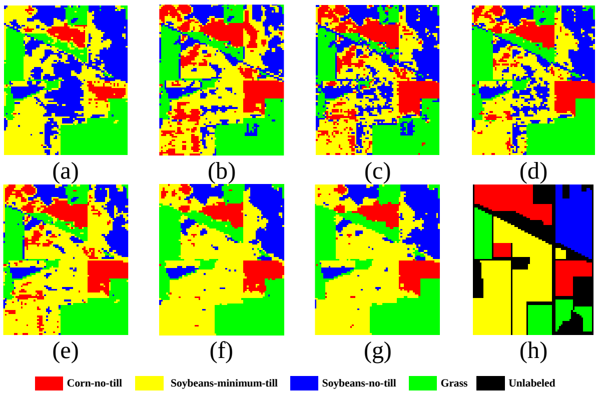

Figure 1. The clustering results on Indian Pines image: (a) CFSFDP, (b) FCM_S1, (C) SSC, (d) S^4C, (e) KSSC, (f) SSC-SMP, (g) KSSC-SMP, (h) ground truth.

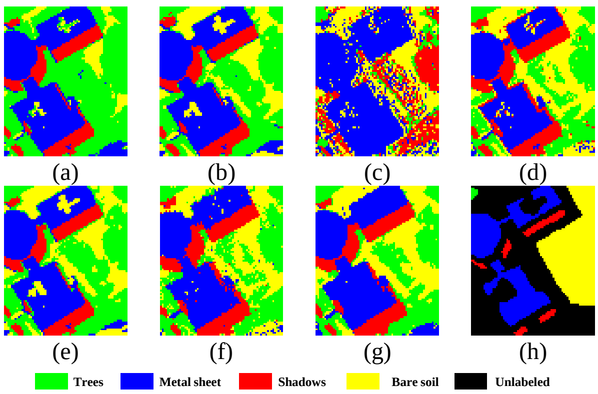

Figure 2. Clustering results on the University of Pavia image: (a) CFSFDP, (b) FCM_S1, (C) SSC, (d) S^4C, (e) KSSC, (f) SSC-SMP, (g) KSSC-SMP, (h) ground truth.

5. CONCLUTION

In this paper, we have proposed a kernel sparse subspace clustering algorithm with spatial max pooling operation (KSSC-SMP) for hyperspectral remote sensing imagery. The proposed approach simultaneously explores the nonlinear structure of HSIs and the wealthy spatial contextual information of the HSIs, which improves on the performance of the classical SSC model to a large degree. The experimental results confirm that the KSSC-SMP algorithm is very competitive in the remote sensing field. However, it still has room for improvement. For instance, the method could be further improved by adaptively determining the regularization parameters and extracting more discriminative spatial features.

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